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ABSTRACT

These appendices to The Irish System of Teacher Education contain: (1) current syllabuses for english and mathematics at the higher and lower levels in the leaving certificate examination; (2) leaving certificate examination marking system; and (3) single honors english degree course syllabus and mathematics degree course syllabus. (JD)

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The System of Teacher Education

Part II

Appendices II, III and IV

Department of Teacher Education
University of Dublin

Series 89/5b

SP 031 455

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The System of Teacher Education

Part II

Appendices II, III and IV

Department of Teacher Education
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Series 89/5b

APPENDIX II

Current syllabuses for English and Mathematics at the Higher and Lower levels in the Leaving Certificate Examination. (Extracted from Rules and Programme for Secondary Schools. Department of Education, Government Stationery Office).

2.—ENGLISH

INTRODUCTION

There are two courses which lead to the Leaving Certificate Examination in English — the Ordinary Course and the Higher Course.

The general aims of these courses are:—

(1) To pursue the fundamental aims of the Intermediate Certificate courses in English at a level appropriate to the needs of senior post-primary pupils.

(2) To help to equip these pupils for life after school, whether at work or in higher education—

(a) by cultivating their powers of communication through language;

(b) by developing their judgement in relation to language, literature and life;

(c) by enriching their experience of life through literature.

The extensive range of literature provided in these courses should be fully exploited. It should be seen as a continuation of Intermediate Certificate literature, demanding continued emphasis upon wide reading and a liberal approach. Its potential for assisting pupils to come to grips with their environment, to widen and deepen their vision of life, and to perfect their mastery of language should be realised. Pupils should be encouraged to take full advantage of the generous freedom of choice in prescribed reading which is offered, and to explore extra-curricular literature under the teacher's guidance. At all times authentic personal response should be encouraged, and sound criteria of literary judgement and good taste should be cultivated. Some introduction to the history of English literature, in relation to the authors selected for study, should be given.

At this level, pupils are expected to have acquired reasonable command of basic language skills in English. Their study of language now should proceed with more advanced work in comprehension and composition. It should also continue to be conducted in close relationship with their study of literature.

Literature is the great repository of language and ideas, the great exemplar of the infinite subtlety of meaning and expression of which language is capable in the hands of the great masters. Therefore, intelligent study of it at this level will offer countless opportunities for developing sensitivity and versatility in the use of language.

This study of language ought to help pupils to acquire a deeper enjoyment and understanding of literature. For example, a writer's style and technique may invite, either orally or in writing, exercises in analysis, discussion, comparison and imitation. Likewise, the content of a piece of writing may offer opportunities for oral and written exercises in comprehension, discussion and summary. The over-all aim to be kept in view should be a fuller experience of the work studied.

It should be noted that formal testing of comprehension in the Higher Course examination will be based upon a set passage of prescribed prose. At Lower Course level the passage set for this purpose will be unprescribed. It will be a passage of well-written, but not difficult, contemporary prose.

A steady output of composition in English should be required from Leaving Certificate pupils, but great flexibility of approach should be permitted. They should be given much freedom of choice from a wide range of subjects to write on. Topics might include personal experience, current affairs, literature, the arts and sciences generally, imaginative excursions, etc. They should also be encouraged to experiment with different forms of composition and different styles of writing. Longer assignments than the essay might sometimes be attempted. Exploration of topics in some depth should be demanded, and a capacity for sustained argument should be developed. Imaginative use of language should be encouraged, but with due regard to correctness of expression. It should be noted that oral composition may be fruitfully practised at this level also, especially in the form of public speaking and debate, or through the medium of improvised drama.

ORDINARY COURSE

PAPER I

Time allowed: 2 hours *Total marks:* 190

Both sections of this paper (*Composition* and *Unprescribed Prose*) must be attempted.

(1) *Prose Composition* (100 *marks*)
A wide choice of subjects will be given.

(2) *Unprescribed Prose* (90 *marks*)
One passage of Unprescribed Prose will be given, and a wide variety of questions will be set on it. These may include –

(i) comprehension questions;
(ii) a summary of all or part of the passage set;
(iii) discussion of the ideas contained in the passage.

A choice of the questions set will be offered.

PAPER II

Time allowed: 3 hours *Total Marks:* 210

All three sections of this paper (*Drama, Poetry* and *Fiction*) must be attempted.
Every section carries 70 marks.

(1) *Drama*
Plays prescribed:–
(i) Hamlet (Shakespeare).
(ii) The Playboy of the Western World (Synge).

A substantial extract (which need not be continuous) will be given from each of the two prescribed plays.
A group of questions will be set on each extract. There will be a choice of questions within each group.
Candidates will be required to deal with *one* play only.

(2) *Poetry*
The following poetry from the *Interim Anthology of Leaving Certificate English Poetry* is prescribed:–

Shakespeare	:	When to the sessions of sweet silent thought. When I have seen by Time's fell hand defaced. No longer mourn for me when I am dead. Was it the proud full sail of his great verse. Poor soul, the centre of my sinful earth.
Jonson	:	A Hymne to God the Father. Song: *from* The Silent Woman.
Vaughan	:	The Retreate. Peace.
Dryden	:	*from* MacFlecknoe.

Shelley	:	Ode to the West Wind. Ozymandias.
Keats	:	Ode to a Nightingale.
Dickinson	:	The five poems in the anthology.
Hopkins	:	The Windhover. Felix Randal. Thou art indeed just, Lord.
Yeats	:	No Second Troy. September 1913. The Fisherman.
Clarke	:	The Lost Heifer. The Blackbird of Derrycairn. The Planter's Daughter.
Kavanagh	:	Stony Grey Soil.

A substantial extract will be given from each of three prescribed poems. (In some cases, the entire poem may be given).

A group of questions will be set on each of these extracts. There will be a choice of questions within each group.

Candidates will be required to deal with *one* poem only.

(3) *Fiction:*
Novels prescribed:—
(i) Persuasion (Austen).
(ii) The Charwoman's Daughter (Stephens).

A substantial extract (which need not be continuous) will be given from each of the two prescribed novels.

A group of questions will be set on each of the two extracts. There will be a choice of questions within each group.

Candidates will be required to deal with *one* novel only.

HIGHER COURSE

PAPER I

Time allowed: 2½ hours. *Total marks:* 160.

Both sections of this paper, 1 and 2, must be attempted.

1. Prose Composition (100 *marks*).
A wide choice of subjects will be given.

2. *Prescribed Prose* (60 marks):

The following passages from *Matriculation and Leaving Certificate Prose* are prescribed:–

No. 1	Of Studies (Bacon)
No. 2	Of Youth and Age (Bacon)
No. 3	Of Simulation and Dissimulation (Bacon)
No. 9	Reflections of the Revolution in France (Burke)
No. 13	The Two Races of Men (Lamb)
No. 14	New Year's Eve (Lamb)
No. 17	The Superannuated Man (Lamb)
No. 20	Hamlet (Hazlitt)
No. 32	Aes Triplex (Stevenson)
No. 33	An Apology for Idlers (Stevenson)

One passage of Prescribed Prose will be given, and a wide variety of questions will be set on it. These may include questions on:–

(i) comprehension;

(ii) comparison/contrast, by reference to material outside the set passage but within the prescribed course;

(iii) analysis of technique, etc.;

(iv) discussion of the prose used in the set passage, and of its author, in relation to the history of non-fictional prose.

A choice of the questions will be offered. Candidates will be required to attempt three of them.

PAPER II

Time Allowed: 3 hours. *Total marks:* 240.

In general essay-type answers will be expected to the questions on this paper.

Every question on this paper carries 60 marks.

Candidates must attempt four questions on this paper, *which must include* – Question A (on Shakespearean tragedy), from Section I (Drama) and one question from Section II (Poetry), and one question from Section III (Fiction).

(1) *Drama*

Plays prescribed:–

(i) Hamlet (Shakespeare).

(ii) Much Ado about Nothing (Shakespeare).

(iii) The Playboy of the Western World (Synge).

On each of the three prescribed plays, a choice of two questions will be offered. The fourth question in the Drama Section, will be of a general nature, but will demand knowledge of any two of the three plays prescribed.

(2) *Poetry:*

The following poetry from the *Interim Anthology of Leaving Certificate English Poetry* is prescribed:–

Shakespeare	:	Any five of the fifteen sonnets in the anthology.
Jonson	:	The three poems in the anthology.
Vaughan	:	The four poems in the anthology.
Milton	:	Paradise Lost, Book I.
Dryden	:	The two poems in the anthology.
Shelley	:	The three poems in the anthology.
Keats	:	The six poems in the anthology.
Dickinson	:	The five poems in the anthology.
Hopkins	:	The six poems in the anthology.
Eliot	:	The Love Song of J. Alfred Prufrock.
Yeats	:	The six poems in the anthology.
Clarke	:	The three poems in the anthology.
Kavanagh	:	The six poems in the anthology.

One specific question will be set on each of four prescribed poets. One of these questions will be of the type appearing on Paper II of the Ordinary Course.

The fifth question in the Poetry Section, will be of a general nature, but will demand close reference to prescribed poets and poems.

(3) *Fiction:*

Novels prescribed:–

(i) Persuasion (Austen).
(ii) Lord Jim (Conrad).
(iii) Any *one* of the following:

Animal Farm (Orwell)
Lord of the Flies (Golding)
The Greatest of These (MacManus)
The Charwoman's Daughter (Stephens)
The Shrimp and the Anemone (Hartley)
A Handful of Dust (Waugh)
The Great Gatsby (Scott Fitzgerald)
The Pearl (Steinbeck)
The Old Man and the Sea (Hemingway)

Three questions will be set.

One specific question will be set on "Persuasion" and one on "Lord Jim".

The third question will be on the modern novel, and will be of a general nature, but it will demand specific reference to one, at least, of the nine novels listed at (iii) above.

Prescribed for the Examination in 1985

Ordinary Course

(1) *Drama:*
(i) King Lear (Shakespeare)
(ii) The Importance of Being Earnest (Oscar Wilde).

(2) *Poetry:* The following poetry from the *Interim Anthology of Leaving Certificate English Poetry* is prescribed:–

Shakespeare	:	When in disgrace with fortune and men's eyes. Not marble, nor the gilded monuments. Since brass, nor stone, nor earth, nor boundless sea. Farewell! thou art too dear for my possessing. My mistress' eyes are nothing like the sun.
Donne	:	The six poems in the anthology.
Herbert	:	The four poems in the anthology.
Milton	:	How soon hath Time. When I Consider.
Wordsworth	:	Loud is the Vale. Surprised by Joy. O Friend! I Know Not.
Keats	:	Terror of Death. Bright Star! Would I were. On the Sea.
Hardy	:	During Wind and Rain. Afterwards.
Thomas	:	Fern Hill.
Yeats	:	No Second Troy. September 1913. The Fisherman.
Kavanagh	:	Stony Grey Soil. Inniskeen Road: July Evening.
Kinsella	:	Mirror in February.

(3) *Fiction:*
Novels prescribed:–
(i) Wuthering Heights (Brontë)
(ii) The Greatest of These (Mac Manus)

PAPER I

Prescribed Prose:

The following passages from *Matriculation and Leaving Certificate Prose* are prescribed:–

No. 7	Iona (Johnson).
No. 8	A Letter to Lord Chesterfield (Johnson).
No. 14	New Year's Eve (Lamb).
No. 17	The Superannuated Man (Lamb).
No. 26	The Isles of the North (Newman).
No. 27	(a) The Value of a University Training (Newman) (b) The Value of a University Training (Newman) (c) The Idea of a University (Newman).
No. 32	Aes Triplex (Stevenson).
No. 33	An Apology for Idlers (Stevenson).

PAPER II

(1) *Drama:*

(i) King Lear (Shakespeare).
(ii) The Tempest (Shakespeare).
(iii) The Importance of Being Earnest (Oscar Wilde).

(2) *Poetry:* The following poetry from the *Interim Anthology of Leaving Certificate English Poetry* is prescribed:–

Chaucer	: The Prologue to the Canterbury Tales (Excerpt)
Shakespeare	: Any five of the fifteen sonnets in the anthology.
Donne	: The six poems in the anthology.
Herbert	: The four poems in the anthology.
Marvell	: Thoughts in a Garden.
Milton	: All of the poems in the anthology except "Paradise Lost", Book I.
Pope	: The Rape of the Lock — all of the excerpts in the anthology.
Wordsworth	: The four poems in the anthology.
Keats	: The six poems in the anthology.
Hardy	: The four poems in the anthology.
Thomas	: The two poems in the anthology.
Yeats	: The six poems in the anthology.
Kavanagh	: The six poems in the anthology.
Kinsella	: The two poems in the anthology.

(3) *Fiction:*

Novels prescribed:–

(i) Wuthering Heights (Brontë)
(ii) Castle Rackrent (Edgeworth).
(iii) Any *one* of the following:–

Animal Farm (Orwell)
Lord of the Flies (Golding)
The Greatest of These (MacManus)
The Charwoman's Daughter (Stephens)
The Shrimp and the Anemone (Hartley)
A Handful of Dust (Waugh)
The Great Gatsby (Scott Fitzgerald)
The Pearl (Steinbeck)
The Old Man and the Sea (Hemingway)

8.–MATHEMATICS

Preamble

The present revised syllabus endeavours to combine in one unified structure topics which are traditional with those which are modern and relevant. Appropriate emphasis is placed on the logical steps which guide students in making decisions and vadid judgments and due consideration is given to the fact that Mathematics is an ever-expanding open-ended subject. Although the sole justification for the inclusion of Mathematics in the curriculum is not the use that is made of it outside school, however the many varied and interesting applications constitute a powerful motivation which is an important factor in the natural process of learning.

In presenting this syllabus to students, the teacher's objectives should be:

- to develop conceptual and meaningful Mathematics together with efficient computational skills;
- to emphasise key concepts and fundamental structures;
- to present Mathematics as an ever-expanding open-ended subject;
- to show Mathematics both as an abstract, autonomous body of knowledge as well as a useful, operational tool;
- to encourage active involvement in enquiry-oriented learning;
- to develop an understanding of elementary statistics and of numerical methods;
- to coordinate with other subjects;
- to enable students to attain knowledge and insight by means of classroom and independent study;
- to prepare students for further study in Mathematics;
- to encourage logical thinking.

In achieving these objectives teachers should endeavour to interest pupils in the subject, to develop in them a love of Mathematics and an appreciation of its method, power and elegance.

Accuracy, neatness of presentation and logical lay-out of solutions should receive sufficient attention as should skill in the use of instruments in geometrical constructions and in the sketching of diagrams and graphs.

It is recommended that reference be made where appropriate to the history of Mathematics and to the lives of great mathematicians.

ORDINARY LEVEL

Mensuration: Regular areas and solids including cone.

Statistics: Elementary idea of frequency and cumulative frequency distributions. Graphical treatment. Median. Simple treatment of standard deviation given an array of numbers or a frequency distribution, including grouped frequencies. (Use of assumed mean not required). Weighted averages.

Equations: Solution of quadratic equations and of cubic equations with at least one integral root. Linear equations in two and in three unknowns, including sets of equations in two unknowns having non-unique solutions. Equations of the form.

$$\frac{3}{3x-2} - \frac{1}{2x+3} = 2 - \frac{x+1}{6x^2+5x-6}$$

Manipulation of formulae.

Factors: Cubic expressions of the form $a^3 + b^3$, $8x^3 - 27$. Use of remainder theorem.

Indices and Logarithms: Use of logarithmic tables (or slide rule) in calculations. (Proofs of formulae not required).

Sequences and Series: Arithmetic and geometric. Derivation of formulae. Summation. Applications to include compound interest formula. Sum of infinite geometric series as the $\lim_{n\to\infty} S_n$.

Relations: Examples of order relations.

Functions: Domain, codomain, range. Bijections, inverse functions, composition of functions.

Graphs: Quadratic and cubic functions. Interpretation as regards changes of value and sign.

Inequalities: Quadratic inequalities in one variable. Linear inequalities in two variables and elementary linear programming limited to two lines and two lines parallel to the axes.

Trigonometry: Radian measure of angles. Definitions of the trigonometric functions sine and cosine for all values of the independent variable. Graphs of sine and cosine to illustrate periodicity. Definition of the tangent function. Compound angle formulae (proofs not required). Simple identities. Area of triangle. Sine and cosine rules and their applications.

Complex Numbers: Addition, subtraction, multiplication and division. Representation by Argand diagram. Absolute value (modulus).

Geometry

Concurrencies in a Triangle: Bisectors of angles, perpendicular bisectors of sides, medians, perpendiculars from vertices to opposite sides. Meaning of centroid and of orthocentre.

Theorem 5.1	The measure of the angle at the centre of a circle is twice the measure of an angle at the circle standing on the same arc.
Deduction	All the angles at the circle standing on the same arc are equal in measure.
Theorem 5.2	Triangles on the same base and between the same parallels are equal in area.
Theorem 5.3	The areas of two triangles of equal height are proportional to the lengths of their bases.
Theorem 5.4	Two sides of a triangle are divided proportionally by a line drawn parallel to the third side.
Theorem 5.5	If the angles of two triangles are equal in measure, then the lengths of their corresponding sides are proportional.
Theorem 5.6	[ab] and [cd] are two chords of a circle. If the lines ab and cd intersect in k, then $\lvert ak \rvert . \lvert kb \rvert = \lvert ck \rvert . \lvert kd \rvert$ (Internal and external cases)
Theorem 5.7	From a point p outside a circle a tangent is drawn to touch the circle at t and a line is drawn to cut the circle at a and b. Then $\lvert pa \rvert . \lvert pb \rvert = \lvert pt \rvert^2$.
Theorem 5.8	In a right angled triangle the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides. (Pythagoras). (**Note:** The converse of each of the above theorems is true. Proof not required).

Analytical Geometry: Coordinates (including the image of a point under a parallel projection and under the isometries listed in the Intermediate Certificate syllabus). Distance between points. Area of triangle. Midpoint of line segment. Slope. Equation of line in the forms $y = mx + c$ and $y - y_1 = m(x - x_1)$. Line through two given points, lines parallel to and lines perpendicular to a given line and through a given point. Intersection of two lines. The equation $x^2 + y^2 = a^2$. Intersection of a line and a circle. Proving a line is a tangent to a circle. The equation of a circle in the form $(x - h)^2 + (y - k)^2 = a^2$.

Vectors: The composition of two translations is a translation. Vector $\vec{ab}$ represented as the translation $\vec{ab}$. Addition and subtraction of vectors. Multiplication of a vector by a scalar. The pointed plane. The perpendicular unit vectors $\vec{i}$, $\vec{j}$.

Binomial Theorem: Meaning of the symbols $n!$, $\binom{n}{r}$. The expansion of $(x + a)^n$ for $n \in \mathbf{N}$. (Proof not required). Easy applications.

Calculus: Rate of change. Geometrical idea of a tangent line. Slope of tangent line defined as a limit. Definition of a derivative as a limit. Calculation of the derivative of polynomials of degree not exceeding three from first principles. Formulae for the derivative of x^n $(n \in N)$, $u + v$, uv, $\frac{u}{v}$ and for the chain rule e.g. $(x^7 + 3)^5$, $(4x^2 - 3x + 1)^2$. (Proofs not required). Simple problems to include the application of the first derivative to drawing graphs of quadratic and of cubic functions. Local maximum and local minimum.

COMPUTER STUDIES

This section is optional. It will not form part of the Leaving Certificate Examination but pupils who take it, and perform satisfactorily, will be given a statement to that effect by the Department. School authorities who wish their pupils to receive such a statement must complete an application form, which may be had from the Department of Education, Post-Primary Branch (Inspection Section), Hawkins House (floor 3), Dublin 2.

As part of the application form schools will be asked to give details of the syllabus they propose to follow. The syllabus should include the following aspects:—

The Development of the computer; Social implications and use in everyday life; Careers in computing; Structured diagrams; Problem analysis; Communicating with the computer; One low level language, where feasible, such as CECIL or CSSP; One high level language such as COMAL; Emphasis on input, control, process, output, testing, branching, loops, functions; Types of input/output devices.

It is envisaged that pupils would spend about 35 hours on this section in the course of one school-year.

Each pupil should write and successfully run about ten programmes. Two or more pupils may undertake joint projects. Records of the work done must be available for monitoring by the Department.

Application forms in respect of the school-year 1983/84 should be returned to the Department in good time, preferably by 31 March 1983.

LEAVING CERTIFICATE PROGRAMME

HIGHER LEVEL

Complex Numbers: *Need for complex numbers. Complex conjugate, modulus. Argand diagram. Meaning of* $|z-a|$ *for* $a \in C$. *Easy loci. Complex roots of* $f(x)=0$, *where* $f(x)$ *is a polynomial with real coefficients. De Moivre's Theorem — proof by induction when* $n \in N$. *Applications when* $n \in Q$.

$|z|^2 = z\bar{z}$; $\quad |z_1 z_2| = |z_1| . |z_2|$;

$|z_1 + z_2| \leq |z_1| + |z_2|$; $\quad |z_1 - z_2| \geq |z_1| - |z_2|$.

Equations: Linear equations and inequalities in two variables. Linear equations in three variables including sets of equations having non-unique solutions. Remainder Theorem and its application to finding the roots of cubic equations. Approximate roots of cubic equations.

Binomial Theorem: Permutations and combinations. Proof by induction of binomial theorem when $n \in N$. Numerical applications when $n \in Q$.

Analytical Geometry (Line and Circle): Distance of a point from a line. Angle between lines. Equation of a line in parametric form. Equation of a line through the intersection of two given lines. Equation of a pair of lines in the form $(a_1 x + b_1 y + c_1)(a_2 x + b_2 y + c_2) = 0$. General equation of a circle. Equation of circle through the points of intersection of (i) two circles (ii) line and circle. Equation of tangent at a point of the circle. Intersection of line and circle.

Trigonometry: Derivation and application of formulae on page 9 of Tables excluding $e^{in\theta}$. Domain, codomain and range of trigonometric functions. Period of a function f as the least value of $\ell > 0$ for which $f(x + \ell) = f(x)$, $x \in R$. The inverse functions $x \to \sin^{-1} x$ and $x \to \tan^{-1} x$.

Matrices: 2 x 2 matrices; addition, multiplication by a scalar. Product of matrices. Inverse of a 2 x 2 matrix. Application of 2 x 2 matrices to the linear transformations:– axial and central symmetry, parallel projection, rotation about origin as centre and their compositions.

Statistics: Sample point (e), sample space (S), event (E). Mutually exclusive events ($E \cap F = \phi$). The probability, P(E), of an event E is a real number satisfying:

Axiom 1:	$P(E) \geq 0$
Axiom 2:	$P(S) = 1, P(\phi) = 0$
Axiom 3:	$P(E) = P(E_1) + P(E_2) + \ldots + P(E_k)$, where $E = E_1 \cup E_2 \ldots \cup E_k$ and where $E_1, E_2 \ldots, E_k$ are singletons.
Theorem S1	If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$

Theorem S2 If E and F are subsets of S, then
$P(E \setminus F) = P(E) - P(E \cap F)$

Theorem S3 If E and F are subsets of S, then
$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Definition E and F are equally likely events $\iff P(E) = P(F)$

Theorem S4 If E is a subset of S, then
$$P(E) = \frac{\#(E)}{\#(S)}$$

Definition E and F are independent events $\iff P(E \cap F) = P(E).P(F)$

Theorem S5 If E and F are independent events, then E and F' are also independent events.

Binomial distribution ($\bar{x} = np$, $\sigma = \sqrt{npq}$) (proof for σ not required).

Standard units $z = \frac{x - \bar{x}}{\sigma}$

Use of Tables on page 36 to test a null hypothesis at the 5% level of significance.

OR

Vectors and Linear Transformations: Vector $\vec{ab}$ represented as the translation $\vec{ab}$. The pointed plane Π_o. The vector $\vec{x}$ is the vector $\vec{ox}$. Addition of vectors as the composition of two translations. Multiplication of a vector by a scalar. Commutative, associative and distributive laws. Length of a vector. Transformation of the plane is a map $\Pi \to \Pi$.
Linear transformation f defined by $f(a\vec{x} + b\vec{y}) = af(\vec{x}) + bf(\vec{y})$.
Orthonormal basis $\vec{i}, \vec{j}$. Scalar product $\vec{x} \cdot \vec{y} = |\vec{x}|\,|\vec{y}| \cos\theta$
and its geometrical meaning.

Theorem V 1 Parallel projection on a line containing the origin is a linear transformation.

Theorem V 2 Scalar product distributes addition: $\vec{x}.(\vec{y} + \vec{z}) = \vec{x}.\vec{y} + \vec{x}.\vec{z}$

Deduction: $\vec{x} = x_1\vec{i} + x_2\vec{j}, \vec{y} = y_1\vec{i} + y_2\vec{j} \Rightarrow \vec{x}.\vec{y} = x_1y_1 + x_2y_2$.

Theorem V 3 If r is a point of a line ab, then
$\vec{r} = t\vec{b} + (1 - t)\vec{a}$, where $t \in R$.

Deduction 1: If $r \in ab$ and $|ar| : |rb| = m : n$, where $m, n \in R$, then

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n} \qquad \text{if } r \in [a\ b]$$

$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m - n} \qquad \text{if } r \notin [a\ b]$$

Deduction 2: If g is the centroid of a triangle abc, then $\vec{g} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$

Theorem V 4 In any triangle abc, the circumcentre, the orthocentre and the centroid are collinear.

Theorem V 5 If a, b, c are any three points such that $m\vec{a} + n\vec{b} + k\vec{c} = \vec{o}$ for all positions of the origin and for m, n, k $\in$ R, then m + n + k = 0.

Analytical Geometry (Parabola): The parabola considered as a locus. The equations $y^2 = \pm 4ax$, $x^2 = \pm 4ay$. Parametric equation: $x = at^2$, $y = 2at$. Tangent to a parabola at a point of the parabola. Intersection of line and parabola. The equations $(y - c)^2 = \pm 4a(x - b)$; $(x - b)^2 = \pm 4a(y - c)$.

OR

Groups: Definition. Commutative groups. Examples of groups under the operations + and × from real numbers, 2 × 2 matrices, residue classes. Examples of groups under composition from the symmetries of an equilateral triangle and square. The group S, Δ where S is the set of all subsets of a given set and Δ is symmetric difference. Recognition of subgroups. Isomorphism of two groups.

Sequences and Series: Evaluation of simple limits assuming the theorems on sums, products and quotients of limits. Evaluation of limits as $x \to a$. Sequences, including arithmetic and geometric with applications to repayment of loans, including use of compound interest formula. Convergence and divergence of sequences. Series including arithmetic and geometric. Convergence and divergence of series. Comparison and ratio tests for series of positive terms.

Calculus: Differentiation: Derivatives from first principles. Rate of change. Derivatives of products, quotients, composite functions (function of a function), polynomials. Local maxima and minima. Points of inflexion. Elementary curve sketching, including parabola and ellipse in standard form. Derivative of (i) trigonometric functions (ii) inverse trigonometric functions $\sin^{-1}$, $\tan^{-1}$ (iii) exponential function (iv) logarithmic function (v) parametric functions of the form $x = f(t)$, $y = g(t)$.

Calculus: Integration: Integration of x^r and of polynomials; trigonometric functions and functions of the form $\sin^2 mx$, $\sin^3 mx$, $\sin mx \cos^2 mx$; the exponential function and functions of the form

$$\frac{1}{a^2 + x^2}, \qquad \frac{1}{\sqrt{a^2 - x^2}}, \qquad \sqrt{a^2 - x^2};$$

product of sines and cosines of multiple angles. Use of substitution. Definite integrals with application to areas and volumes of revolution.

Note: Statistics is an alternative to Vectors and Linear Transformations. Analytical Geometry (Parabola) is an alternative to Groups.

Computer Studies. As for Ordinary Level.

LEAVING CERTIFICATE PROGRAMME

NOTES ON THE REVISED SYLLABUS

These notes deal with: 1 **Statistics** and 2 **Vectors and linear transformations.**

1. STATISTICS

Sample Space (S): the set of all possible outcomes of an experiment.
Event (E): subset of S
Sample Point (e): an outcome of an experiment
Singleton (E_i): the event containing the singleton e_i.
Mutually Exclusive Events: $E \cap F = \phi$

Example

A multiple choice test consists of five questions and to each question there are four possible answers – (a), (b), (c), (d).

An outcome (e_i) of this test might be (a), (d), (a), (a), (c), where (a) is given as the solution to Q 1, (d) to Q 2, (a) to Q 3, (a) to Q 4, (c) to Q 5.

In the Venn diagram each dot represents an outcome of the form (a), (d), (a), (a), (c).

In all there are 4^5 possible outcomes which is the sample space, S.

If E is the event: (a) is the answer given to each of the first four questions, and F is the event: (b) is the answer given to each of the first four questions, then E and F each consist of 4 sample points and since they have no point in common, they are mutually exclusive.

However if H is the event: (a) is the answer given to each of the first two questions then one sample point common to E and H is (a), (a), (a), (a), (a) so that E and H are not mutually exclusive.

The Axioms of Probability

To every $E \subset S$ there is a real number P(E), called the probability of E, satisfying

Axiom 1: $P(E) \geq 0$

Axiom 2: $P(S) = 1$, $P(\phi) = 0$

Axiom 3: If $E = E_1 \cup E_2 \cup \ldots \cup E_k$, where $E_1, E_2, \ldots, E_k$ are singletons, then $P(E) = P(E_1) + P(E_2) + \ldots + P(E_k)$.

Theorem 1

If E and F are mutually exclusive events, then

$$P(E \cup F) = P(E) + P(F)$$

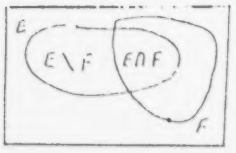
Proof: Let $E = E_1 \cup E_2 \cup ... \cup E_k$
and $F = F_1 \cup F_2 \cup ... \cup F_t$
where $E_1, E_2 ... F_1, F_2 ...$ are singletons, then
$E \cup F = E_1 \cup E_2 \cup ... \cup E_k \cup F_1 \cup F_2 \cup ... \cup F_t$.
And so by Axiom 3.

$$P(E \cup F) = P(E_1) + P(E_2) + ... + P(F_1) + P(F_2) + ...$$
$$= P(E) + P(F)$$

q.e.d.

Theorem 2

If $E \subset S$ and $F \subset S$,
then
$P(E \setminus F) = P(E) - P(E \cap F)$.



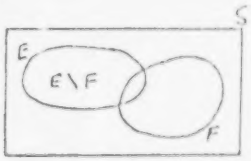
Proof: $E \setminus F$ and $E \cap F$ are mutually exclusive events and

$$E = (E \setminus F) \cup (E \cap F).$$
$$\text{Hence } P(E) = P(E \setminus F) + P(E \cap F)$$
$$\text{and so } P(E \setminus F) = P(E) - P(E \cap F).$$

q.e.d.

Theorem 3

If $E \subset S$ and $F \subset S$,
then
$P(E \cup F) = P(E) + P(F) - P(E \cap F)$.



Proof: $E \setminus F$ and F are mutually exclusive events and

$$E \cup F = (E \setminus F) \cup F.$$
$$\text{Hence } P(E \cup F) = P(E \setminus F) + P(F)$$
$$= P(E) - P(E \cap F) + P(F).$$

q.e.d.

Equally Likely Events

Definition: E and F are equally likely $\iff P(E) = P(F)$

Theorem 4.

If $E \subset S$, then $P(E) = \frac{\#(E)}{\#(S)}$

Proof: Let $S = E_1 \cup E_2 \cup \ldots \ldots \cup E_n$ so that $\#(S) = n$
and $E = E_1 \cup E_2 \cup \ldots \cup E_r$ so that $\#(E) = r$.
Assume the singletons are equally likely and that $r \leqslant n$.

Then $P(S) = P(E_1) + P(E_2) + \ldots + P(E_n)$
so that $1 = n\,P(E_1) \Rightarrow P(E_1) = \frac{1}{n}$.

Hence $P(E) = r\,P(E_1) = r\left(\frac{1}{n}\right) = \frac{r}{n} = \frac{\#(E)}{\#(S)}$

q.e.d.

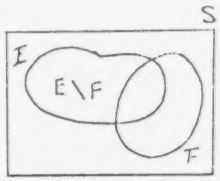
Independent Events

Definition: E and F are independent $\Longleftrightarrow P(E \cap F) = P(E).P(F)$

Theorem 5

E and F independent $\Rightarrow$ E and F′ independent

Proof: $E \cap F' = E \setminus F$



Hence
$P(E \cap F') = P(E \setminus F)$
$= P(E) - P(E \cap F)$
$= P(E) - P(E).P(F)$
$= P(E)\,[1 - P(F)]$
$= P(E).P(F')$ since $S = F \cup F' \Rightarrow 1 = P(F) + P(F')$ … by Th. 1.

q.e.d.

Mean of a Binomial Distribution

The binomial distribution applies when the result of an experiment is either a success or a failure. The probability of a success is usually denoted by p and the probability of a failure by q so that

$$P(\text{success}) = p \text{ and } P(\text{failure}) = q.$$

In the case of n trials there can be 0, 1, 2, ..., n successes. If we denote by x the number of successes, then x can be 0, 1, 2, ...n.. The variable is called a RANDOM VARIABLE and the probabilities for x = 0, 1, 2, ..., n are given by the terms of the binomial expansion $(q + p)^n$ so that

$P(x = 0) = q^n$; $P(x = 1) = \binom{n}{1} pq^{n-1}$;

$P(x = 2) = \binom{n}{2} p^2q^{n-2}$; $P(x = r) = \binom{n}{r} p^rq^{n-r}$;

$P(x = n) = p^n$.

Example 1.

One quarter of the number of items manufactured by a factory are faulty. As a quality control four items are chosen at random. How many of these should one expect to be good?

Solution:

Let the random variable, x, denote the number of good items, then the probabilities for x = 0, 1, 2, 3, 4 are given by the terms of the binomial expansion $(¼ + ¾)^4$ and so the resulting probability distribution is

x	0	1	2	3	4
P(x)	$(¼)^4$	$\binom{4}{1}(¾)(¼)^3$	$\binom{4}{2}(¾)^2(¼)^2$	$\binom{4}{3}(¾)^3(¼)$	$(¾)^4$

Calculating the mean of this distribution as if it was a frequency distribution we get

$$\bar{x} = \frac{\sum x\,P(x)}{\sum P(x)} = \frac{0\left(\frac{1}{4}\right)^4 + 1\binom{4}{1}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^3 + \cdots + 4\left(\frac{3}{4}\right)^4}{\left(\frac{1}{4}\right)^4 + \binom{4}{1}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^3 + \cdots + \left(\frac{3}{4}\right)^4}$$

$$= \left(\frac{1}{4}\right)^4\left[0 + 12 + 108 + 324 + 324\right]$$

$$= \frac{768}{256} = 3.$$

Hence we would expect to get 3 good items among the four. In other words if this experiment is repeated many times, the mean number of good items tends to 3.

Definition

$\bar{x}$, the mean of a probability distribution, is defined as

$$\bar{x} = \sum_{x=0}^{n} x\,P(x)$$

where $x = 0, 1, 2, \ldots, n$ gives the possible number of successes and $P(x)$ is the probability of x successes.

Theorem

In the case of the binomial probability distribution, $(q + p)^n$

$$\bar{x} = np,$$

where $\bar{x}$ is the mean number of successes in n trials.

Proof:

$$\bar{x} = \sum_{x=0}^{n} x\,P(x) = 0\,P(0) + 1\,P(1) + 2\,P(2) + \ldots + n\,P(n)$$

$$= 0.q^n + 1\binom{n}{1}pq^{n-1} + 2\binom{n}{2}p^2q^{n-2} + \ldots + np^n$$

$$= np\left[q^{n-1} + \binom{n-1}{1}pq^{n-2} + \ldots + p^{n-1}\right]$$

$$= np\,(q + p)^{n-1}$$

$$= np, \text{ since } (q + p) = 1.$$

q.e.d.

Standard deviation of a binomial distribution

Let σ denote the standard deviation of the binomial probability distribution $(q + p)^n$.

Then

$$\sigma = \sqrt{npq}$$

Standard Units

Standard units are of the form $\frac{x - \bar{x}}{\sigma}$, where $\bar{x}$ is the mean and σ the standard deviation. Standard units are used to compare two sets of data.

Example 2

The marks of five candidates A, B, C, D, E in two tests are given in the table:

	A	B	C	D	E
Test 1	50	53	60	66	71
Test 2	8	10	12	14	16

In which test did D do best?

Solution:

In order to express the marks in standard units we first calculate $\bar{x}$ and σ for each test.

Test 1: $\bar{x} = \dfrac{50 + 53 + \ldots + 71}{5} = 60.$

$$\sigma = \sqrt{\frac{(50-60)^2 + \ldots + (71-60)^2}{5}}$$

$$= \sqrt{\frac{100 + 49 + 0 + 36 + 121}{5}} = \sqrt{61.2} = 7.823$$

Hence the standard mark corresponding to 66 is

$$\frac{x - \bar{x}}{\sigma} = \frac{66 - 60}{7.823} = 0.77 \text{ to two places of decimals}$$

Test 2: $\bar{x} = \dfrac{8 + 10 + \ldots + 16}{5} = 12.$

$$\sigma = \sqrt{\frac{(8-12)^2 + \ldots + (16-12)^2}{5}}$$

$$= \sqrt{\frac{16 + 4 + 0 + 4 + 16}{5}} = \sqrt{8} = 2.828$$

Hence the standard mark corresponding to 14 is

$$\frac{x - \bar{x}}{\sigma} = \frac{14 - 12}{2.828} = 0.71 \text{ to two places of decimals.}$$

We conclude that D did better in test 1 than in test 2.

To be able to compare various distributions the random variable x is transformed into standard units, z, by the linear transformation

$$x \to \frac{x - \bar{x}}{\sigma} = z.$$

This new random variable z has mean 0 and standard deviation 1. To see this, let x take the n values $x_1, x_2, \ldots, x_n$. Then

$$\text{mean of } z = \frac{1}{n} \sum_{r=1}^{n} \frac{x_r - \bar{x}}{\sigma} = \frac{1}{\sigma} \sum_{r=1}^{n} \frac{x_r}{n} - \frac{1}{n\sigma} \sum_{r=1}^{n} \bar{x}$$

$$= \frac{\bar{x}}{\sigma} - \frac{n\bar{x}}{n\sigma} = 0,$$

$$\text{and standard deviation of } z = \sqrt{\frac{1}{n} \sum_{r=1}^{n} \left(0 - \frac{x_r - \bar{x}}{\sigma}\right)^2}$$

$$= \sqrt{\frac{1}{\sigma^2} \sum_{r=1}^{n} \left(\frac{(x_r - \bar{x})^2}{n}\right)} = \sqrt{\frac{1}{\sigma^2} \sigma^2} = 1.$$

From the Binomial Probability Distribution to the Normal Probability Distribution

If a random variable x has a binomial probability distribution, then for large n the random variable

$$z = \frac{x - \bar{x}}{\sigma}$$

approximates to the Normal Probability Distribution given by

$$P(z \leq z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2} \, dz.$$

The equation of the normal curve is

$$y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

and its shape is shown on p. 36 of the Tables. A property of the normal distribution is that the area under the normal curve between the z – axis and the lines $z = z_1$ and $z = z_2$ is the probability that the random variable z lies between z_1 and z_2.

Example 3

If z is a random variable which has a normal probability distribution, find the probability that

$$\text{(i) } z \geqslant 2.0 \quad \text{(ii) } z \leqslant -3.0 \quad \text{(iii) } 1.5 \leqslant z \leqslant 2.5.$$

Solution:

The probabilities are given by the areas under the normal curve and these areas are on p. 36 of the Tables.

(i) Area for $z \geqslant 2$ = 1 – (area for $z \leqslant 2$) = 1 – 0.9772.
= 0.0228,

the required probability.

(ii) Since the normal curve is symmetrical about the y-axis, the area for $z \leqslant -3.0$ = area for $z \geqslant 3.0$
= 1 – (area for $z \leqslant 3.0$) = 1 – 0.99865
= 0.00135.

the required probability.

(iii) For $z \leqslant 2.5$ the area is 0.99379
For $z \leqslant 1.5$ the area is 0.9332.
Hence the area for $1.5 \leqslant z \leqslant 2.5$ is 0.99379 – 0.9332.
= 0.06059.

the required probability.

Example 4

x is a random variable having mean 125 and standard deviation 10. If x has a binomial distribution, estimate the probability that $x \geqslant 141$

Solution:

Since x is binomially distributed

$$z = \frac{x - \bar{x}}{\sigma} = \frac{x - 125}{10}$$

approximates to a normal distribution. Hence

$$x \geq 141 \Rightarrow Z \geq \frac{141 - 125}{10} = 1.6.$$

The area for $z \geq 1.6 = 1 - (\text{area for } z \leq 1.6)$

$= 1 - 0.9452$

$= \underline{0.0548}$

the required estimate.

Example 5

When seeds of a certain type are sown, the probability of a seed not germinating is ¼. When 300 seeds are sown, estimate the probability that between 70 and 80 seeds will not germinate.

Solution:

Either a seed germinates or it does not. Hence if the random variable x is the number of seeds that do not germinate, then x is binomially distributed so that

$$\bar{x} = np = \frac{300}{4} = 75$$

$$\text{and } \sigma = \sqrt{npq} = \sqrt{\frac{300.3}{16}} = \frac{30}{4} = 7.5$$

$$\text{Hence } z = \frac{x - 75}{7.5} \text{ approximates to a normal distribution.}$$

$$\text{Hence } 70 \leq x \leq 80 \Rightarrow \frac{70 - 75}{7.5} \leq z \leq \frac{80 - 75}{7.5}$$

$$\Rightarrow -0.67 \leq z \leq 0.67$$

The area for $-0.67 \leq z \leq 0.67$

$= 1 - (\text{area for } z \leq -0.67 + \text{area for } z \geq 0.67)$

$= 1 - (2 \text{ area for } z \geq 0.67)$

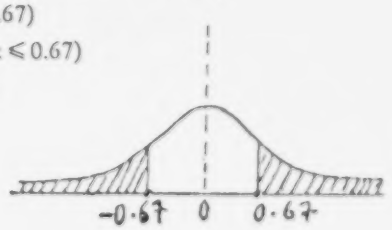
$= 1 - 2\ (1 - \text{area for } z \leq 0.67)$

$= 1 - 2\ (1 - 0.7486)$

$= 1 - 2\ (0.2514)$

$= 1 - 0.5028$

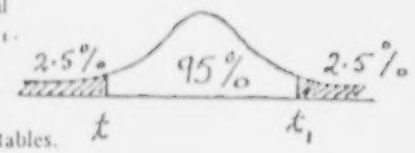
$= \underline{0.4972}.$



the required probability.

Null Hypothesis – Significance

95% of the area under the normal curve lies between $z = t$ and $z = t_1$.
To find t and t_1 we have:



Area for $z \leqslant t_1 = 0.975$

$\Rightarrow t_1 = 1.96$ from the tables.

By symmetry $t = -1.96$.

This means that if x is a random variable which is normally distributed, there is a probability of 95% that

$$(\bar{x} - 1.96\,\sigma) \leqslant x \leqslant (\bar{x} + 1.96\,\sigma).$$

If an experiment results in a value of x (assuming normal distribution) which is greater than $\bar{x} + 1.96\,\sigma$, we conclude that factors other than chance are involved – that some form of bias is present. We consequently say that the result of the test or experiment is SIGNIFICANT at the 5% LEVEL OF SIGNIFICANCE. The probability of getting a value of x outside the domain $[\bar{x} - 1.96\,\sigma,\ x + 1.96\,\sigma]$ is 5%. In other words ASSUMING NO BIAS, BY HYPOTHESIS, there is only a small chance (i.e. 5%) that the value of x is outside $[\bar{x} - 1.96\,\sigma, \bar{x} + 1.96\,\sigma]$ and so we decide that on the result of the experiment that there is BIAS. This decision is going to be incorrect once in every 20 experiments. The hypothesis that there is no bias is called the NULL HYPOTHESIS.

Example 7

A penny is tossed 900 times and comes up head 480 times. Does this result indicate that the penny is biased?

Solution:

Our null hypothesis is that the penny is not biased. On this assumption or hypothesis we calculate z. If $z \geqslant 1.96$, we say the penny is biased.

Let x be the random variable indicating the possible number of heads obtained in 900 tosses. Then x has a binomial distribution and so for large n

$$z = \frac{x - \bar{x}}{\sigma} = \frac{x - np}{\sqrt{npq}} = \frac{x - 900\,(\tfrac{1}{2})}{\sqrt{900.\,\tfrac{1}{2}.\,\tfrac{1}{2}}} = \frac{x - 450}{15}$$

is normally distributed, approximately.

$x = 480 \Rightarrow z = 2 > 1.96.$

We say that this result is significant at the 5% level of significance and so conclude that there is evidence of bias in the penny.

Note:

If 20 pennies are picked at random and each is tossed 900 times resulting in 480 heads for each penny, then we would expect that 19 pennies were biased and one was not.

Example 8

An unbiased coin is tossed 576 times and it is necessary to predict with an accuracy of 95% the limits between which the number of heads will lie. Estimate these limits.

Solution:

Let x be the random variable denoting the possible number of heads resulting from 576 tosses of the coin. Then x has a binomial distribution with mean and standard deviation given by

$$\bar{x} = np = \frac{576}{2} = 288 \text{ and}$$

$$\sigma = \sqrt{npq} = \sqrt{\frac{576}{4}} = 12.$$

$$\text{Let } z = \frac{x - 288}{12}.$$

Then z is normally distributed, approximately.

Probability of 95% $\Rightarrow$ Area is 95%

$$\Rightarrow -1.96 \leqslant z \leqslant 1.96$$

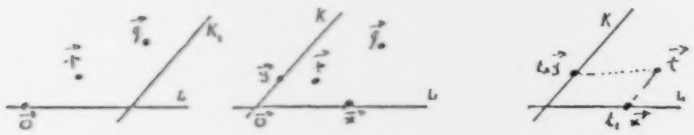
$$\Rightarrow -1.96 \leqslant \frac{x - 288}{12} \leqslant 1.96$$

$$\Rightarrow 264.48 \leqslant x \leqslant 311.58$$

And so an estimate of the required limits for x are 265 and 311

2. VECTORS AND LINEAR TRANSFORMATIONS

Theorem 6.1 **Parallel projection on a line containing the origin is a linear transformation**



Let L be the line containing the origin and let the projection, f, be parallel to K_1. Let $\vec{p}$ and $\vec{q}$ be any two vectors.

To prove: (i) $f(\vec{p}+\vec{q}) = f(\vec{p}) + f(\vec{q})$

(ii) $f(\lambda\vec{p}) = \lambda f(\vec{p})$ for $\lambda \in R$

Construction: Let $K \parallel K_1$ be a line through o and take $\vec{x}$ and $\vec{y}$ as basis, as in diagram. Let $\vec{p} = p_1\vec{x} + p_2\vec{y}$ and $\vec{q} = q_1\vec{x} + q_2\vec{y}$ so that $\vec{p}+\vec{q} = (p_1+q_1)\vec{x} + (p_2+q_2)\vec{y}$.

Proof: Let $\vec{t} = t_1\vec{x} + t_2\vec{y}$ be any vector. Then the image of $\vec{t}$ by f is $t_1\vec{x}$ i.e. $f(\vec{t}) = t_1\vec{x}$.

Hence $f(\vec{p}+\vec{q}) = (p_1+q_1)\vec{x}$

and $f(\vec{p}) = p_1\vec{x}$; $f(\vec{q}) = q_1\vec{x}$.

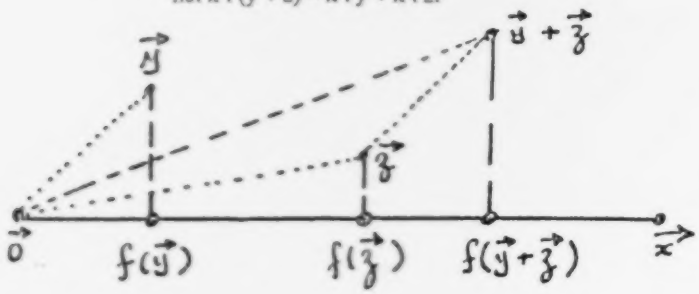
Since $(p_1+q_1)\vec{x} = p_1\vec{x} + q_1\vec{x}$, we have

$f(\vec{p}+\vec{q}) = f(\vec{p}) + f(\vec{q})$.

Again $f(\lambda\vec{p}) = \lambda p_1\vec{x} = \lambda f(\vec{p})$. q.e.d.

Theorem 6.2 **Scalar product distributes addition**

i.e. $\vec{x}\,.\,(\vec{y}+\vec{z}) = \vec{x}\,.\,\vec{y} + \vec{x}\,.\,\vec{z}$.



Let $\vec{x}, \vec{y}, \vec{z}$ be any three vectors and let f be the orthogonal projection of the plane on the line ox so that $f(\vec{y}), f(\vec{z}), f(\vec{y}+\vec{z})$ are the images of the vectors $\vec{y}$, $\vec{z}$, $(\vec{y}+\vec{z})$ under f.

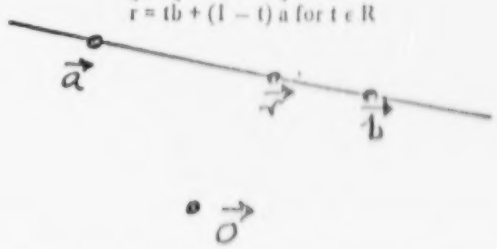
To prove: $\vec{x} \cdot (\vec{y}+\vec{z}) = \vec{x} \cdot \vec{y} + \vec{x} \cdot \vec{z}$

Proof:
$$\begin{aligned}\vec{x} \cdot (\vec{y}+\vec{z}) &= \vec{x} \cdot f(\vec{y}+\vec{z}) \\ &= \vec{x} \cdot [f(\vec{y}) + f(\vec{z})] \quad \ldots\ldots \text{Th. 6.1} \\ &= |\vec{x}| \; (|f(\vec{y}) + f(\vec{z})|) \\ &= |\vec{x}| \cdot (|f(\vec{y})| + |f(\vec{z})|) \\ &\qquad \text{since } \vec{o}, f(\vec{y}), f(\vec{z}) \text{ collinear} \\ &= |\vec{x}| \cdot |f(\vec{y})| + |\vec{x}| \cdot |f(\vec{z})| \\ &= \vec{x} \cdot \vec{y} + \vec{x} \cdot \vec{z}\end{aligned}$$

q.e.d.

Deduction:
$$\begin{aligned}\vec{x} \cdot \vec{y} &= (x_1 \vec{i} + x_2 \vec{j}) \cdot (y_1 \vec{i} + y_2 \vec{j}) \\ &= x_1 y_1 + x_2 y_2\end{aligned}$$
since $\vec{i}.\vec{i} = 1 = \vec{j}.\vec{j}$ and $\vec{i}.\vec{j} = 0$.

Theorem 6.3 If r is a point of a line ab, then
$$\vec{r} = t\vec{b} + (1-t)\vec{a} \text{ for } t \in R$$



Let $\vec{a}$ and $\vec{b}$ be any two vectors and let o be the origin. Let r be any point in the line ab.

Proof:
$$\begin{aligned}\vec{r} &= \vec{a} + t(\vec{b} - \vec{a}) \text{ for } t \in R \\ &= t\vec{b} + (1-t)\vec{a}.\end{aligned}$$

q.e.d.

Deduction 1; If $r \in ab$ and $|ar| : |rb| = m : n$, where $m, n \in R^+$, then

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n} \qquad \text{if } r \in [ab]$$

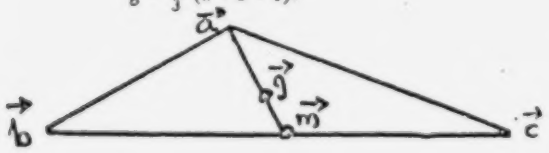
$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m - n} \qquad \text{if } r \notin [ab]$$

Proof: In the first case let $t = \frac{m}{m + n}$ which ensures $r \in [ab]$ and so $1 - t = \frac{n}{m + n}$. In the second case let $t = \frac{m}{m - n}$ which ensures that $r \notin [ab]$ and so $1 - t = \frac{-n}{m - n}$.

q.e.d.

Deduction 2: If $\dot{g}$ is the centroid of a triangle abc, then

$$\vec{g} = \tfrac{1}{3}(\vec{a} + \vec{b} + \vec{c}).$$



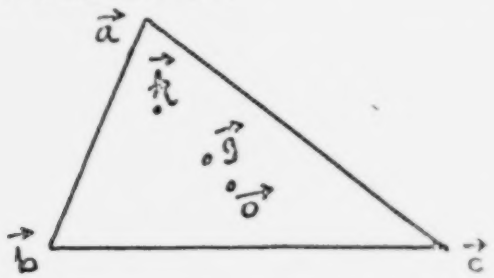
Proof: $\vec{g} = \frac{2\vec{m} + \vec{a}}{3}$ ..... $|ag| : |gm| = 2 : 1$

$= \frac{2}{3}\left(\frac{\vec{b} + \vec{c}}{2}\right) + \frac{1}{3}\vec{a}$ ..... $|bm| : |mc| = 1 : 1$

$= \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$.

q.e.d.

Theorem 6.4 In any triangle abc, the circumcentre, the orthocentre and the centroid are collinear.



Let $\vec{o}$, the circumcentre, be taken as origin and let $\vec{k} = \vec{a} + \vec{b} + \vec{c}$.

Proof: $(\vec{k} - \vec{a}) \cdot (\vec{b} - \vec{c}) = (\vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c}) = |\vec{b}|^2 - |\vec{c}|^2 = 0.$

Hence ka ⊥ bc.

Similarly $(\vec{k} - \vec{b}) \cdot (\vec{a} - \vec{c}) = 0 \Rightarrow$ kb ⊥ ac

and $(\vec{k} - \vec{c}) \cdot (\vec{a} - \vec{b}) = 0 \Rightarrow$ kc ⊥ ab.

Hence $\vec{k}$ is the orthocentre of the triangle abc.

Let g be the centroid of the △ abc so that $\vec{g} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$.

Then $\vec{o}$, $(\vec{a} + \vec{b} + \vec{c})$, $\frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ are collinear.

q.e.d.

(**Note:** The line containing the circumcentre, the orthocentre and the centroid is called Euler's line. Euler, a Swiss Mathematician lived from 1707 to 1783.)

Theorem 6.5 If a, b, c are any three points such that

$$m\vec{a} + n\vec{b} + k\vec{c} = \vec{oo}$$

for all positions of the origin and for m, n, k, ∈ R, then m + n + k = 0.

Let $\vec{a}$ be any vector w.r.t. an origin o and let o_1 be any other origin.

Then, taking o as origin, $\vec{a} = \vec{o}_1 + \vec{o_1a}$ so that $\vec{o_1a} = \vec{a} - \vec{o}_1$.

Hence $\vec{a}$ w.r.t. the origin o is $\vec{a} - \vec{o}_1$ w.r.t. the origin o_1.

Similarly $\vec{b}$ w.r.t. the origin o is $\vec{b} - \vec{o}_1$ w.r.t. the origin o_1

and $\vec{c}$ w.r.t. the origin o is $\vec{c} - \vec{o}_1$ w.r.t. the origin o_1.

To Prove: $m + n + k = 0$

Proof: Since by hypothesis the linear relation

$$m\vec{a} + n\vec{b} + k\vec{c} = \vec{oo}$$

holds for all positions of the origin, then w.r.t. o_1 this relation is

$$m(\vec{a} - \vec{o}_1) + n(\vec{b} - \vec{o}_1) + k(\vec{c} - \vec{o}_1) = \vec{o_1o_1} = \vec{oo}$$

Hence $m\vec{a} + n\vec{b} + k\vec{c} = (m + n + k)\vec{o}_1$

$$\Rightarrow \quad \vec{oo} = (m + n + k)\vec{o}_1$$

$$\Rightarrow m + n + k = 0$$

q.e.d.

APPENDIX III

Leaving Certificate Examination Mark Scheme.

Results at the Leaving Certificate Examination will be shown in the form of grades, each grade representing a percentage range of marks as follows:

Grade	Percentage Range
A	85 or over
B	70 but less than 85
C	55 but less than 70
D	40 but less than 55
E	25 but less than 40
F	10 but less than 25
No Grade	Less than 10

APPENDIX IV

A Single Honours English degree course syllabus

B Mathematics degree course syllabus.

School of English

The School of English offers the single honor course in English described below and two-subject moderatorship courses in medieval and renaissance English and modern English (for details see TWO-SUBJECT MODERATORSHIP COURSES) which cannot be combined with each other.

SINGLE HONOR COURSE IN ENGLISH

This course covers English literature from Old English to the present day, including Anglo-Irish and American literature. The Freshman years provide a grounding in major authors and genres through courses based on selected texts. Students also take courses in practical criticism and in medieval and renaissance critical theory in the Freshman years, and the early periods of the language are studied to provide a background for the reading of literary texts and a foundation for those wishing to specialise in this area in the Sophister years. Teaching is by lectures and tutorials. In the Sophister years students choose their programmes of study from a number of optional courses. Teaching is by seminar and/or lecture. An optional elementary course in Latin through reading is also provided.

Junior Freshmen

1 Old English language and literature
2 Drama to Marlowe
3 The short poem from Chaucer to Milton
4 Seventeenth and eighteenth-century poetry
5 Drama from Jonson to the present
6 The novel
7 Practical criticism

Senior Freshmen

1 Chaucer and Spenser
2 Langland and the *Gawain* poet
3 Shakespeare
4 Romantic, Victorian and modern poetry
5 Anglo-Irish literature
6 American literature
7 Medieval and renaissance critical theory

Sophister options

Options are offered to students in their Sophister years in the following areas, subject to availability of staff in any given year.

1 *Beowulf*
2 Old English literature
3 Middle English literature
4 Medieval and renaissance drama[1]
5 Romance
6 Sixteenth-century literature
7 Shakespeare

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[1] this option is provided by the Samuel Beckett Centre for Drama and Theatre Studies.

8 Milton
9 Seventeenth-century literature
10 Eighteenth-century literature
11 Romantic poetry
12 Nineteenth-century poetry
13 Nineteenth-century fiction
14 Modern poetry
15 Twentieth-century fiction
16 Anglo-Irish literature
17 American literature
18 Theory of narrative
19 Modern European drama[1]
20 History of criticism
21 Intellectual history
22 Creative writing
23 The English language
24 Old Norse

Students choose four course options in each of the Sophister years.

N.B. The same options cannot be taken in both Sophister years.

Details of optional courses and regulations relating to them may be obtained from the school secretary.

Junior Freshman assessment

The annual examination in the Junior Freshman year consists of six 3-hour papers of equal weight:

1 Old English language and literature
2 Drama to Marlowe
3 The short poem from Chaucer to Milton
4 Seventeenth and eighteenth-century poetry
5 Drama from Jonson to the present
6 The novel

A proportion of marks equivalent to one paper is allotted to work presented during the year in relation to the course on practical criticism and an equal proportion of marks is allotted to term essays in relation to courses 1, 2 and 3.

Senior Freshman assessment

The annual examination in the Senior Freshman year consists of six 3-hour examination papers of equal weight:

1 Chaucer and Spenser
2 Langland and the *Gawain* poet
3 Shakespeare
4 Romantic, Victorian and modern poetry
5 Anglo-Irish literature
6 American literature

Candidates are required to sit papers 1-4 and *either* paper 5 *or* paper 6. In relation to the course in which they do not choose to sit an examination they must submit a 3,000-5,000 word essay by the first day of Trinity term. This essay is allotted a proportion of marks equivalent to one paper in the annual examination.

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[1]This option is provided by the Samuel Beckett Centre for Drama and Theatre Studies.

An equal proportion of marks is allotted to work presented during the year in relation to the course on medieval and renaissance critical theory.

Moderatorship examination, part I, papers, essays, marks

Moderatorship part I is held at the end of the Junior Sophister year and consists of three 3-hour papers of equal weight, one on each of three of the course options studied in the Junior Sophister year. Candidates also submit two essays on topics related to a fourth course option studied in the Junior Sophister year, which taken together are equal in weight to one paper in moderatorship (certain language-based options may not be assessed in this way). The maximum number of marks allotted to moderatorship part I is 400.

Moderatorship examination, part II, papers, essays, marks

Moderatorship part II is held at the end of the Senior Sophister year and consists of four 3-hour papers of equal weight, one in each of three course options studied in the Senior Sophister year and a general paper. Candidates also submit two essays on topics related to a fourth course in the Senior Sophister year which taken together are equal in weight to one paper in moderatorship or, with the permission of the school committee, may substitute a dissertation for these two essays (certain language-based options may not be assessed this way). The maximum number of marks allotted to moderatorship part II is 600.

II SCHOOL OF MATHEMATICS

1 The School of Mathematics offers a degree course in mathematics in which students can select, particularly in the Sophister years, combinations of courses chosen from various areas; these include pure mathematics, applied mathematics, theoretical physics, numerical analysis, computing and statistics.

2 Mathematics may also be studied as a component of a two-subject moderatorship course in combination with one of several other subjects.

Fees

3 See COLLEGE CHARGES.

Rowe Fund Library

4 Members of the Dublin University Mathematical Society have the use of a mathematical library in the society's rooms, 39 Trinity College, in addition to the College library.

5 In the Junior Freshman year students take courses 111, 121, 131, 141, 151 and 162.[2] Annual examinations are held at the end of Trinity term with supplemental examinations in September. One paper is set on each of the courses 121, 131, 141, 151 and 162. There is, in addition, an informal test on the work of the first term. This is held at the beginning of the Hilary term to advise students on their suitability for the course.

6 In the Senior Freshman year students take an algebra course 211/213, an analysis course 221/223, a mathematical methods course 231 and three other courses chosen from courses 212, 241, 251, 261, 262.

In each Sophister year students take five group III courses. In the Junior Sophister year, students may take the two group II courses they omitted in the Senior Freshman year as a substitute for one group III course.

In all years students should make their choice of courses in consultation with the course advisers in mathematics; in the Sophister years constraints may be imposed on the choice of courses and each student's choice must be approved by a course adviser. Change of choice is not permitted after the end of the Hilary term. With the consent of a course adviser, students are permitted to take one extra group III course in each Sophister year. If an extra course is presented for examination then one group III course will be ignored in calculating the overall mark for the year.

7 In each year of the course the annual examinations are held in May/June with supplementals — for Freshmen only — in September.

To gain a pass in the examinations of the Freshman years, students must either pass in each subject or compensate for F_1 grades in at most two subjects by their level of performance in the others. Junior and Senior Freshman students who have failed in the Trinity term examinations must take supplemental examinations, in those subjects in which they have failed, at the beginning of Michaelmas term. For regulations governing repetition of year see §8 below. To proceed to the Senior Sophister year, a student must pass in at least three group III (or two group III and two group II) courses and achieve an overall average of at least 40 per cent at the Junior Sophister examination.

Junior Sophister students who marginally fail their annual examination may choose to graduate with an ordinary degree in mathematics or may apply to repeat their year in accordance with §8 below.

The examinations of the two Sophister years count equally towards the overall mark for moderatorship. Sophister students may be required to submit a short thesis on some subject.

Students who fail to reach moderatorship standard in their Senior Sophister year will be awarded an ordinary B.A. degree.

Repetition of year in mathematics

8 Students may apply for permission, or may be required, to repeat an academic year as provided under general College regulations. Students are not allowed to repeat a year if they have obtained an overall grade of F_3 for the year.

Option in mathematics and computing

9 This is a special option within the mathematics course to cater for students with a strong interest in computer science but who wish to pursue this from a mathematical

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[1]Students who entered the school before October 1988 are subject to the regulations shown in the *Calendar* of their year of entry.

[2]For description of numbered courses see list of courses in §14, pp. P5–6.

viewpoint while simultaneously studying mathematics to a high level. Students taking this option take courses 111, 121, 131, 141, 151 and 162 in the Junior Freshman year. In the Senior Freshman year they take courses 211, 212, 221, 231, 261 and 262. In each of the Sophister years five courses are chosen from a range of (group III) options in mathematics and computer science, including 311, 321, 361, 362, 363, 371, 372, 373, 461, 462.

SPECIAL HONOR COURSE IN THEORETICAL PHYSICS

10 In addition to the honor course in mathematics, students may study mathematics as a principal subject in the special honor course in theoretical physics. This is one of the subject options within the natural sciences moderatorship and consists of courses and experimental work in the Department of Pure and Applied Physics together with courses in mathematics and theoretical physics in the School of Mathematics. In the Junior Freshman year students take the course and practical work in physics as required in the School of Natural Sciences together with the following mathematics courses: 111, 121, 131, 141 and 151. They also attend the lectures of course I in chemistry (Junior Freshman natural sciences) in the Michaelmas term.

In their Senior Freshman year they take the course and practical work in physics together with courses 211, 221 (excluding measure and Lebesgue integration), 231 and 241 in mathematics. In the Junior Sophister year they take specified courses and practical work in physics together with specified courses in applied mathematics/theoretical physics. In the Senior Sophister year practical work in physics is discontinued and students take specified courses in physics together with specified courses in applied mathematics/theoretical physics.[1]

Students reading theoretical physics take their physics and mathematics examinations in May/June. Further regulations are set out under section III, SCHOOL OF NATURAL SCIENCES.

Double moderatorship in mathematics and experimental physics

11 Junior Sophister students reading theoretical or experimental physics who show exceptional ability and promise may, provided they have completed the requisite mathematics courses in their Freshman years to entitle them to transfer to mathematics, apply to the Dean for permission to proceed in experimental physics and mathematics simultaneously. The Dean will only grant permission if, in the judgement of the Head of Department of Pure and Applied Mathematics and the Head of Department of Pure and Applied Physics, candidates have a reasonable expectation of a first class honor result in both degree examinations.

TWO-SUBJECT MODERATORSHIP COURSES

12 Students may study mathematics as a principal subject in the two-subject moderatorship course in philosophy and mathematics. In this course students take a number of courses in each of the two subjects, specialising to some extent in one subject in their Sophister years. For full details see TWO-SUBJECT MODERATORSHIP COURSES.

In addition students may also study mathematics in one of the two-subject moderatorship courses, for details see TWO-SUBJECT MODERATORSHIP COURSES. As a three year subject it can be combined with a wide range of arts subjects. As a four year subject it can be combined with economics, geography, psychology or sociology.

Transfer of course

13 Students may apply through their tutor to transfer from the special honor course in theoretical physics to the honor course in mathematics not later than the end of the Junior

[1]See also SCHOOL OF NATURAL SCIENCES, section III, §§ 10–13.

Sophister year.

Students may apply through their tutor to transfer from any two-subject moderatorship course involving mathematics to the honor course in mathematics not later than the end of the Junior Freshman year.

Courses

14 In this list of courses prerequisites are shown in brackets ( ).

Group I

111 Sets and maps; group practice (isometries, symmetries, permutations, modular arithmetic); group theory (group axioms, homomorphisms, group action, sub-groups, Lagrange's theorem, product groups, quotient groups, Sylow theorems); group representations

121 The number system; elementary number theory; sequences and series; functions of one variable; uniform convergence; elementary topology

131 Applied linear algebra; simple modelling; linear waves; vector analysis; partial differentiation; maxima and minima; multiple integration; determinants and matrices

141 Vector algebra; Newton's laws of motion; particle dynamics

151 Programming in a high level language; introduction to statistics, concepts of statistical inference and related probability theory, elementary statistical methods and applications, statistical software; an introduction to the theory of computation

162 Introduction to computer systems and their applications; problem solving and the development of algorithms; programming in a high level language; introduction to systems software

Group II

211 Linear algebra; vector spaces; inner product spaces; multilinear algebra; functions of a complex variable; contour integration (111, 121, 131)

212 Metric spaces; topology (111, 121)

213 Linear algebra; vector spaces; inner product spaces; linear algebra with applications; functions of a complex variable; contour integration (111, 121, 131)

221 Measure and Lebesgue integration; functions of several variables; differential forms (111, 121, 131)

223 The derivative; mean value theorems; the Riemann integral; differential calculus of scalar and vector fields; Taylor's theorem; extrema; line integrals; surface integrals; multiple integrals (111, 121, 131)

231 Lagrangian methods, curvilinear coordinates, Fourier series and Fourier transforms, potential theory (131, 141)

241 Quantum mechanics using Dirac's notation, Hamiltonian dynamics, special relativity (141)

251 Probability theory; distribution theory; estimation and hypothesis testing (151)

261 Introduction to numerical analysis, numerical linear algebra (131)

262 Computer hardware; program design methodologies; data structures; searching and sorting; file organisation; application of computing in engineering and engineering software packages (162)

Group III

311 Abstract algebra

312 Discrete mathematics

Faculty of Science

321 Modern analysis
341 Non-linear systems I
342 Simple mathematical models in physics (for Junior Sophister theoretical and experimental physics students only)
343 Topics in theoretical physics (for theoretical physics students only)
344 Non-linear systems II
361[1] Topics in theoretical computing I
362[2] Advanced programming; compiler construction; data base management (162, 262)
363[1] Topics in theoretical computing II
371[1] Topics in theoretical computing III
372[2] Microprocessor architecture; assembly language programming; introduction to operating systems and file organisation (162, 262)
373[1] Topics in theoretical computing IV
411 Functional analysis
412 Probability theory
413 Partial differential equations
414 Complex analysis
415 Topics in analysis
421 Topology
422 Galois theory
423 Tensors
424 Group representations
425 Differential geometry
431 Fluid mechanics
432 Electromagnetic theory
433 Statistical mechanics
441 Quantum mechanics
442 General relativity
443 Statistical physics
451 Probability and statistics I
452 Probability and statistics II
461 Numerical solution of differential equations I
462 Numerical solution of differential equations II

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[1]These may include the following: abstract data types, coding theory, complexity, computability, encryption, formal specification, information theory, Lambda calculus, linear programming, symbolic manipulation, synergetics and theorem proving.

[2]Students are advised to take course 372 in the Junior Sophister year and course 362 in the Senior Sophister year.